Section 12.3

Odds

Odds Against

- $$\text{Odds against event} = \frac{P(\text{event fails to occur})}{P(\text{event occurs})} = \frac{P(\text{failure})}{P(\text{success})}$$

Example: Odds Against

- Example: Find the odds against rolling a 5 on one roll of a die.

- $P(5) = \frac{1}{6}$ $\qquad P(\text{fails to roll a 5}) = \frac{5}{6}$

  odds against rolling a 5 $= \dfrac{\frac{5}{6}}{\frac{1}{6}} = \frac{5}{6} \cdot \frac{6}{1} = \frac{5}{1}$

  The odds against rolling a 5 are 5:1.

Odds in Favor

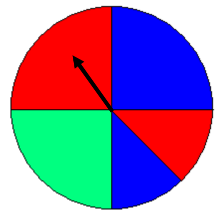
- Odds in favor of event $= \dfrac{P(\text{event occurs})}{P(\text{event fails to occur})}$

  $$= \frac{P(\text{success})}{P(\text{failure})}$$

Example

- Find the odds in favor of landing on blue in one spin of the spinner.

$P(\text{blue}) = \frac{3}{8}$ $\quad P(\text{not blue}) = \frac{5}{8}$



$$\text{odds in favor} = \frac{\frac{3}{8}}{\frac{5}{8}} = \frac{3}{8} \cdot \frac{8}{5} = \frac{3}{5}$$

The odds in favor of spinning blue are 3:5.

Probability from Odds

- Example: The odds against spinning a blue on a certain spinner are 4:3. Find the probability that
- a) a blue is spun.
- b) a blue is not spun.

Solution

- Since the odds are 4:3 the denominators must be 4 + 3 = 7.
- The probabilities ratios are:

$$P(\text{blue}) = \frac{3}{7}$$

$$P(\text{not blue}) = \frac{4}{7}$$

Section 12.4

Expected Value (Expectation)

Expected Value

- $$E = P_1 \cdot A_1 + P_2 \cdot A_2 + P_3 \cdot A_3 + \ldots + P_n \cdot A_n$$
- The symbol P_1 represents the probability that the first event will occur, and A_1 represents the net amount won or lost if the first event occurs.
- P_2 is the probability of the second event, and A_2 is the net amount won or lost if the second event occurs.
- And so on…

Example

- Teresa is taking a multiple-choice test in which there are four possible answers for each question. The instructor indicated that she will be awarded 3 points for each correct answer and she will lose 1 point for each incorrect answer and no points will be awarded or subtracted for answers left blank.
  - If Teresa does not know the correct answer to a question, is it to her advantage or disadvantage to guess?
  - If she can eliminate one of the possible choices, is it to her advantage or disadvantage to guess at the answer?

Solution

- Expected value if Teresa guesses.

$$P(\text{guesses correctly}) = \frac{1}{4}$$

$$P(\text{guesses incorrectly}) = \frac{3}{4}$$

$$E = \frac{1}{4}(3) + \frac{3}{4}(-1)$$

$$= \frac{3}{4} - \frac{3}{4} = 0$$

- Therefore, over the long run, Theresa will neither gain nor lose points by guessing.

Solution (continued) —eliminate a choice

- $$P(\text{guesses correctly}) = \frac{1}{3}$$

$$P(\text{guesses incorrectly}) = \frac{2}{3}$$

$$E = \frac{1}{3}(3) + \frac{2}{3}(-1)$$

$$= 1 - \frac{2}{3} = \frac{1}{3}$$

- Therefore, over the long run, Theresa will, on average, gain $^1/_3$ point each time she guesses when she can eliminate one choice.

Example: Winning a Prize

- When Calvin Winters attends a tree farm event, he is given a free ticket for the $75 door prize. A total of 150 tickets will be given out. Determine his expectation of winning the door prize.
- $$E = \frac{1}{150}(75) + \frac{149}{150}(0)$$
  $$= \frac{1}{2}$$
- Therefore, Calvin's expectation is $0.50, or 50 cents.

Example

- When Calvin Winters attends a tree farm event, he is given the opportunity to purchase a ticket for the $75 door prize. The cost of the ticket is $3, and 150 tickets will be sold. Determine Calvin's expectation if he purchases one ticket.

Solution

$$E = \frac{1}{150}(72) + \frac{149}{150}(-3)$$
$$= \frac{72}{150} - \frac{447}{150}$$
$$= -\frac{375}{150}$$
$$= -2.50$$

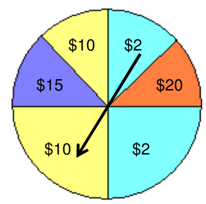
- Calvin's expectation is –$2.50 when he purchases one ticket.

Fair Price

Fair price = expected value + cost to play

Example

- Suppose you are playing a game in which you spin the pointer shown in the figure, and you are awarded the amount shown under the pointer. If it costs \$10 to play the game, determine:
- a) the expectation of the person who plays the game.
- b) the fair price to play the game.



Solution

Amt. Shown on Wheel	**\$2**	**\$10**	**\$15**	**\$20**
Probability	3/8	3/8	1/8	1/8
Amount Won/Lost	–\$8	\$0	\$5	\$10

- $$E = \frac{3}{8}(-\$8) + \frac{3}{8}(\$0) + \frac{1}{8}(\$5) + \frac{1}{8}(\$10)$$
$$= \frac{-24}{8} + 0 + \frac{5}{8} + \frac{10}{8}$$
$$= \frac{-9}{8} = -1.125 \approx -\$1.13$$

Solution

- Fair price = expectation + cost to play
  = –$1.13 + $10
  = $8.87

Thus, the fair price is about $8.87.